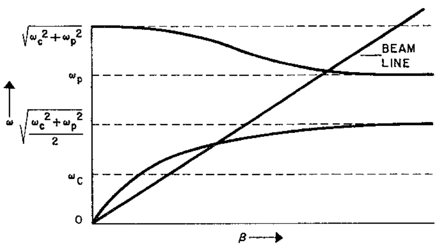
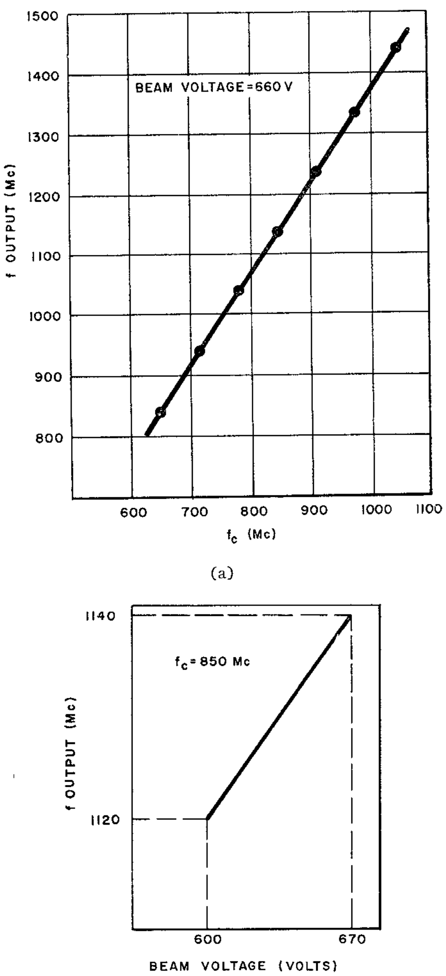


Fig. 3—ω-β curve for angularly symmetric mode.



(a)

(b)

Fig. 4—Variations of output frequency.

formed analytically, using the determinantal equation given by Trivelpiece.[3])

At frequencies above ω_c, Trivelpiece[3] describes the mode as a surface wave, since charge builds up at the edge of the plasma and field discontinuities exist accordingly. In the surface wave region, the axial electric field is finite on the axis, and increases radially to the plasma edge. It then discontinuously falls to zero at the conducting wall. The coupling impedance on the axis of the plasma has been calculated to be slightly less than 1500 ohms for the actual operating parameters of this experiment.

The oscillation mechanism can be explained in terms of reflections from objects situated along the plasma column. This is verified by the discreteness of the oscillation frequencies shown in Fig. 4(a). This graph shows that as the cyclotron frequency is varied, oscillations can only occur when the reflected wave adds constructively to the forward wave. Fig. 4(b) shows the variation of oscillation frequency with beam voltage when the cyclotron frequency is held constant. It is continuous because the range of oscillation frequencies is very small, and the entire line actually corresponds to a single point in Fig. 4(a). The increase of frequency for increasing voltage is seemingly incongruous with the voltage tuning characteristic of Fig. 3, but it should be remembered that ω_p varies with beam voltage for beam-generated plasma systems. Thus, for increased voltage, the upper cutoff frequency also increases, and the intersection of the beam line with the new ω-β curve will occur at a slightly higher frequency.

In summary, it is seen that an electron beam interacts very strongly with the surface wave mode of a plasma column; this interaction converts dc beam energy into narrow-band RF energy in the plasma.

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[3] A. W. Trivelpiece, "Slow-Wave Propagation in Plasma Waveguides," C.I.T. Electron Tube and Microwave Lab., California Inst. of Technology, Pasadena, Calif., Tech. Rept. No. 7; 1958.

A Double-Prism Attenuator for Millimeter Waves*

A double-prism attenuator, similar to the one built by Garnham[1] for use at 35 Gc, was built in the laboratory for use at 100 Gc and above. This attenuator utilizes the well-known fact that a wave which is incident on the interface between two dielectrics at an angle greater than the critical angle is totally reflected. If the wave is in a medium of index of refraction n and the other medium is free space the critical angle is

$$\theta_c = \sin^{-1}\frac{1}{n}.$$

If another medium of the same refractive index n is placed at a distance d from the first interface (Fig. 1), energy will be transferred into this new medium. The amount of coupled energy is a function of the distance d in terms of the wavelength.

For the case of a plane wave incident on the interface (the dielectric sheets extending to infinity), the amount of energy coupled to the new medium can be obtained analytically. Here we just state the results which are identical to Garnham's.

When the electric field is parallel to the plane of incidence

$$\left|\frac{E_t}{E_i}\right|^2 = \frac{4}{4\cosh^2\alpha d + \dfrac{[(\epsilon+1)-(\epsilon^2+1)\sin^2\theta]^2\sinh^2\alpha d}{\epsilon(\epsilon\sin^2\theta-1)\cos^2\theta}} \tag{1}$$

$$\left|\frac{E_r}{E_i}\right|^2 = \frac{(\epsilon-1)^2(\epsilon\sin^2\theta-\cos^2\theta)^2\sinh^2\alpha d}{4\epsilon\cos^2\theta(\epsilon\sin^2\theta-1)\cosh^2\alpha d+[(\epsilon+1)-(\epsilon^2+1)\sin^2\theta]^2\sinh^2\alpha d}, \tag{2}$$

where

$$\alpha = \frac{2\pi}{\lambda}(\epsilon\sin^2\theta-1)^{1/2}.$$

There are corresponding equations for the case of perpendicular polarization.

A laboratory model of the double-prism attenuator was made of polystyrene. Each prism was half of a 2 inch cube (Fig. 2). The prism tolerance was ±0.0015 inch.

The two prisms were mounted on a brass stand; one was movable and the other fixed. The base of the movable prism slid on two steel rods, its movement being controlled by a micrometer head. Fig. 3 shows the actual attenuator.

Although we realized that the matching of the outer surfaces was a factor necessary in obtaining good agreement between experimental and theoretical results, we neglected doing so because it would restrict the attenuator to a limited frequency range. If the surfaces are not perfectly flat and

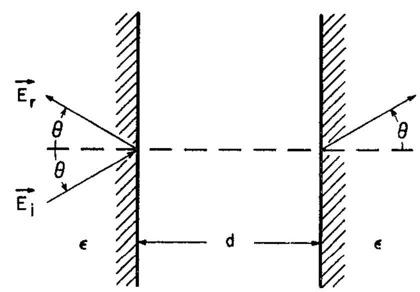

Fig. 1—Double interface arrangement.

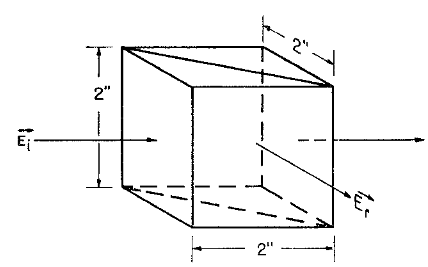

Fig. 2.—Double-prism attenuator.

* Received April 25, 1962. This research was supported by the Air Force Systems Command, United States Air Force.

[1] R. H. Garnham, "Optical and Quasi-Optical Transmission Techniques and Components Systems for Millimeter Wavelengths," Royal Radar Establishment, GT. Malvern, Eng., Rept. No. 3020; March, 1959.

parallel another error is introduced which will increase with frequency.

The response of the detector used in the attenuator measurements followed a square law. This was determined by use of a variable attenuator. This attenuator had been carefully calibrated against a Golay cell detector by the substitution method. The theoretical results and the measured points for one polarization in the 140 Gc and 210 Gc regions are shown in Figs. 4 and 5. The theoretical results were obtained from (1) and (2) using 2.52 as the dielectric constant of polystyrene[2] and 45° as the value for θ. The measured points of the transmitted and reflected wave for parallel polarization were within 1 db of the theoretical values. The difference can be attributed to the following: the lack of matched surfaces, surface tolerances on the coupling surfaces, and diffraction effects.

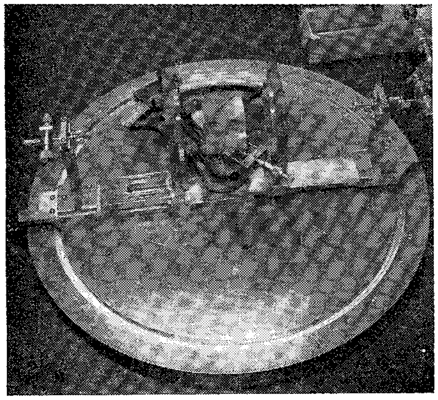

Fig. 3.—Photograph of double-prism attenuator.

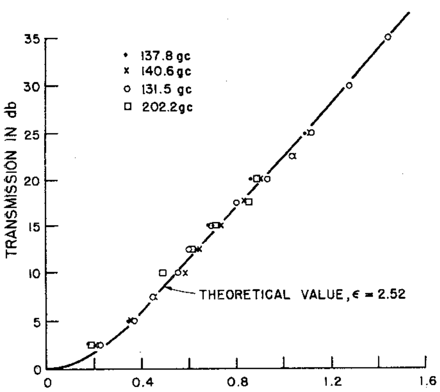

Fig. 4.—Transmission as a function of d/λ for E parallel.

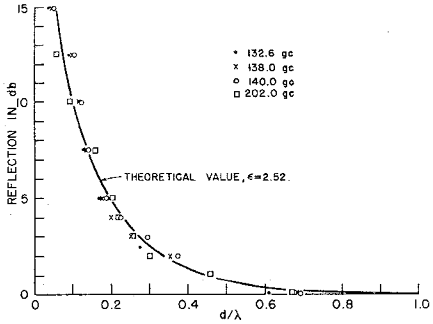

Fig. 5.—Reflection as a function of d/λ for E parallel.

[2] J. C. Wiltse, *et al.*, "Quasi-Optical Components and Surface Waveguides for the 100 to 300 Kmc Frequency Range," Electronic Communications, Inc., Timonium, Md., Sci. Rept. No. 2; November, 1960.

In conclusion, we can say that it is possible to build a double-prism attenuator for millimeter wave applications with good performance. Such a device can also be used as a variable coupler. If greater precision is desired in the attenuator, the outer surfaces of the prisms should be matched into free space; tolerances of less than ±0.0015 inch must be kept and the effects of diffraction should be included.

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Application of Sampling Theorem to the Synthesis of Transmission Line Tapers and Antenna Radiation Patterns*

In this note it is demonstrated how the sampling theorem may be used to reduce the problems of transmission line taper synthesis and antenna aperture field synthesis to that of optimizing a polynomial. In both cases the problem may be stated mathematically in the following form: find the function $g(x)$ which is zero outside the domain $-\pi \leq x \leq \pi$ that will yield a desired $f(u)$ where $f(u)$ is given by the Fourier transform of $g(x)$, *i.e.*,

$$f(u) = \int_{-\pi}^{\pi} g(x) e^{ixu} dx. \tag{1}$$

In the transmission line taper problem $g(x)$ is proportional to the derivative of the logarithm of the taper impedance and $f(u)$ is proportional to the reflection coefficient, as a function of frequency, at the taper input. It is assumed that the taper impedance varies slowly enough so that the square of the reflection coefficient can be neglected.[1,2] In the antenna problem $g(x)$ is proportional to the aperture field for the one dimensional case and $f(u)$ is proportional to the radiation pattern.[3]

Let $g(x)$ be expressed as the Fourier series

$$g(x) = \sum_{n=-\infty}^{\infty} c_n e^{jnx} \tag{2}$$

* Received April 25, 1962.

[1] F. Bolinder "Fourier transforms in the theory of inhomogenous transmission lines," Proc. IRE (*Correspondence*, vol. 38, p. 1354; November, 1950.

[2] R. E. Collin, "The optimum transmission line matching section," Proc. IRE, vol. 44, pp. 539–549; April, 1956.

[3] T. T. Taylor, "Design of line-source antennas for narrow beamwidth and low side lobes," IRE Trans. on Antennas and Propagation, vol. AP-3, pp. 16–28; January, 1955.

From (1) it is found that

$$f(u) = 2\pi \sum_{n=-\infty}^{\infty} c_n \frac{\sin \pi(u-n)}{\pi(u-n)} \tag{3a}$$

$$f(n) = 2\pi c_n \tag{3b}$$

which is the sampling theorem. An arbitrary $f(u)$ would, in general, require a $g(x)$ which is nonzero over $-\infty < x < \infty$. If, however, a set of coefficients was determined by (3b) then the resultant $f(u)$ generated by this $g(x)$ which is zero outside the domain $-\pi \leq x \leq \pi$ would be equal to the arbitrarily specified $f(u)$ at the sampling points and deviate by some finite amount in between. The practical problem is thus seen to be the one specifying an $f(u)$ that can be produced by a $g(x)$ which is zero outside the domain $-\pi \leq x \leq \pi$. This may be done as follows.

Eq. (3a) may be written as

$$f(u) = 2\pi \sum_{n=-\infty}^{\infty} c_n \frac{(-1)^n \sin \pi u}{\pi(u-n)} = 2\pi \frac{\sin \pi u}{\pi u} \sum_{n=-\infty}^{\infty} c_n (-1)^n \frac{u}{u-n}. \tag{4}$$

In most practical cases it is desirable to limit the expansion of $g(x)$ to a finite number of terms, say $2N+1$ terms. From (3b) it is seen that this may be accomplished by specifying $f(u)$ to have zeroes at all but $2N+1$ integer values of u. For convenience it will be assumed here that $f(u)=0$ for $|u| = n > N$. Then (4) becomes

$$f(u) = 2\pi \frac{\sin \pi u}{\pi u} \sum_{n=-N}^{N} c_n (-1)^n \frac{u}{u-n}. \tag{5}$$

This expression is recognized as the partial fraction expansion of some function of the form

$$P(u) \Big/ \prod_{n=1}^{N} (u^2 - n^2)$$

where $P(u)$ is an arbitrary polynomial of degree $2N$ in u, *i.e.*,

$$\begin{aligned} \frac{P(u)}{\prod_{n=1}^{N} (u^2-n^2)} &= \sum_{m=-N}^{N}{}' \frac{P(m)}{(u-m)2m \prod_{\substack{n=1 \\ n\neq m}}^{N} (m^2-n^2)} + \lim_{u\to\infty} \frac{P(u)}{u^{2N}} \\ &= \frac{uP(u)}{u \prod_{n=1}^{N} (u^2-n^2)} \\ &= \sum_{m=-N}^{N}{}' \frac{uP(m)}{(u-m)2m^2 \prod_{\substack{n=1 \\ n\neq m}}^{n} (m^2-n^2)} \\ &\quad + \frac{P(0)}{\prod_{n=1}^{N} (-n^2)} \end{aligned} \tag{6}$$

where the prime means omission of the term $m=0$. Thus (5) becomes

$$f(u) = 2\pi \frac{\sin \pi u}{\pi u} \frac{P(u)}{\prod_{n=1}^{N} (u^2-n^2)}. \tag{7}$$

Any $f(u)$ of this form can be generated by a $g(x)$ which is zero outside the finite domain